4. EXAMPLES

LEARNING OBJECTIVE

1. Are there any convenient functional forms for analyzing consumer choice?

The Cobb-Douglas utility function comes in the form $u(x, y) = x^{\alpha} y^{1-\alpha}$. Since utility is zero if either of the goods is zero, we see that a consumer with Cobb-Douglas preferences will always buy some of each good. The marginal rate of substitution for Cobb-Douglas utility is

$$-\left.\frac{dy}{dx}\right|_{u=u_0} = \frac{\partial u / \partial x}{\partial u / \partial y} = \frac{\alpha y}{(1-\alpha)x}.$$

Thus, the consumer's utility maximization problem yields

$$\frac{p_X}{p_Y} = -\left.\frac{dy}{dx}\right|_{u=u_0} = \frac{\partial u / \partial x}{\partial u / \partial y} = \frac{\alpha y}{(1-\alpha)x}.$$

Thus, using the budget constraint, $(1-\alpha)x p_X = \alpha y p_Y = \alpha(M - x p_X)$. This yields

$$x = \frac{\alpha M}{p_X}, \quad y = \frac{(1-\alpha)M}{p_Y}$$

The Cobb-Douglas utility results in constant expenditure shares. No matter what the price of *X* or *Y*, the expenditure xpX on *X* is αM. Similarly, the expenditure on *Y* is $(1 - \alpha)M$. This makes the Cobb-Douglas utility very useful for computing examples and homework exercises.

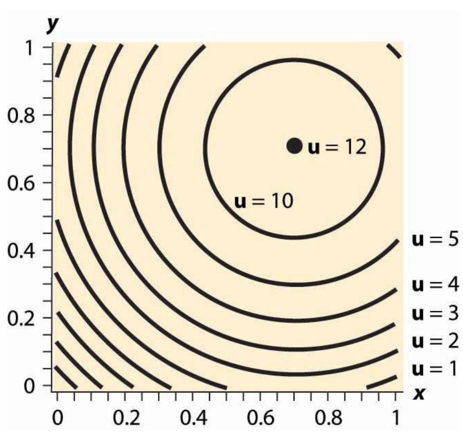
When two goods are perfect complements, they are consumed proportionately. The utility that gives rise to perfect complements is in the form $u(x, y) = \min\{x, \beta y\}$ for some constant β (the Greek letter "beta"). First observe that, with perfect complements, consumers will buy in such a way that $x = \beta y$. The reason is that, if $x > \beta y$, some expenditure on *x* is a waste since it brings in no additional utility; and the consumer gets higher utility by decreasing *x* and increasing *y*. This lets us define a "composite good" which involves buying some amount *y* of *Y* and also buying βy of *X*. The price of this composite commodity is $\beta pX + pY$, and it produces utility $u = \frac{M}{\beta p_X + p_Y}$. In this way, perfect complements boil down to a single good problem.

If the only two goods available in the world were pizza and beer, it is likely that **satiation**–the point at which increased consumption does not increase utility–would set in at some point. How many pizzas can you eat per month? How much beer can you drink? [Don't answer that.]

Satiation

The point at which increased consumption does not increase utility.

FIGURE 4.1 Isoquants for a bliss point



Bliss point

A point that maximizes utility.

What does satiation mean for isoquants? It means there is a point that maximizes utility, which economists call a **bliss point**. An example is illustrated in Figure 4.1. Near the origin, the isoquants behave as before. However, as one gets full of pizza and beer, a point of maximum value is reached, illustrated by a large black dot. What does satiation mean for the theory? First, if the bliss point isn't within reach, the theory behaves as before. With a bliss point within reach, consumption will stop at the bliss point. A feasible bliss point entails having a zero value of money. There may be people with a zero value of money, but even very wealthy people, who reach satiation in goods that they personally consume, often like to do other things with the wealth and appear not to have reached satiation overall.

KEY TAKEAWAYS

- The Cobb-Douglas utility results in constant expenditure shares.
- When two goods are perfect complements, they are consumed proportionately. Perfect complements boil down to a single good problem.
- A bliss point, or satiation, is a point at which further increases in consumption reduce utility.