5. SUBSTITUTION EFFECTS

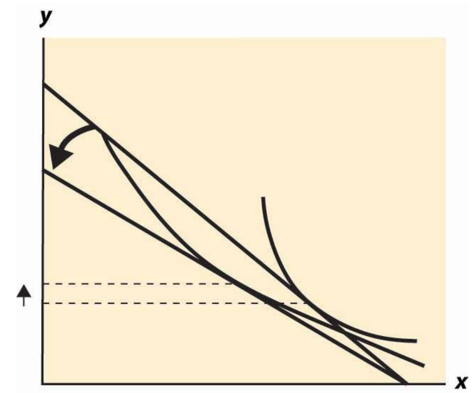
LEARNING OBJECTIVE

1. **When prices change, how do consumers change their behavior?**

It would be a simpler world if an increase in the price of a good always entailed buying less of it. Alas, it isn't so, as Figure 5.1 illustrates. In this figure, an increase in the price of *Y* causes the budget line to pivot around the intersection on the x-axis, since the amount of X that can be purchased hasn't changed. In this case, the quantity y of Y demanded rises.

At first glance, this increase in the consumption of a good in response to a price increase sounds implausible, but there are examples where it makes sense. The primary example is leisure. As wages rise, the cost of leisure (forgone wages) rises. But as people feel wealthier, they choose to work fewer hours. The other examples given, which are hotly debated in the "tempest in a teapot" kind of way, involve people subsisting on a good like potatoes but occasionally buying meat. When the price of potatoes rises, they can no longer afford meat and buy even more potatoes than before.

FIGURE 5.1 Substitution with an increase in price



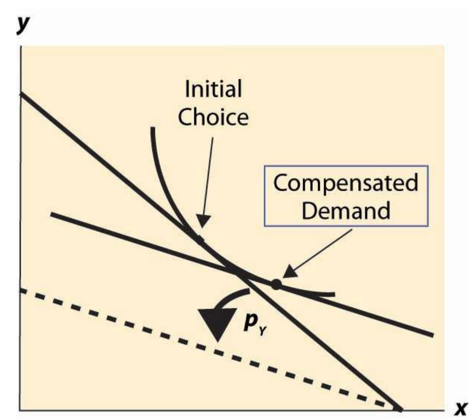
Thus, the logical starting point on substitution—what happens to the demand for a good when the price of that good increases—does not lead to a useful theory. As a result, economists have devised an alternative approach based on the following logic. An increase in the price of a good is really a composition of two effects: An increase in the relative price of the good, and a decrease in the purchasing power of money. As a result, it is useful to examine these two effects separately. The **substitution effect** considers the change in the relative price, with a sufficient change in income to keep the consumer on the same utility isoquant.[2] The income effect changes only income.

To graphically illustrate the substitution effect, consider Figure 5.2. The starting point is the tangency between the isoquant and the budget line, denoted with a diamond shape and labeled "Initial Choice." The price of Y rises, pivoting the budget line inward. The new budget line is illustrated with a heavy, dashed line. To find the substitution effect, increase income from the dashed line until the original isoquant is reached. Increases in income shift the budget line out in a fashion parallel to the original. We reach the original isoquant at a point labeled with a small circle, a point sometimes called the **compensated demand** because we have compensated the consumer for the price increase by increasing income just enough to leave her unharmed, on the same isoquant. The substitution effect is just the difference between these points—the substitution in response to the price change, holding constant the utility of the consumer.

Substitution effect

The effect on consumption of a change in the relative price, with a sufficient change in income to keep the consumer on the same utility isoquant.

FIGURE 5.2 Substitution effect



We can readily see that the substitution effect of a price increase in Y is to decrease the consumption of Y and increase the consumption of X.[3] The **income effect** is the change in consumption resulting from the change in income. The effect of any change in price can be decomposed into the substitution effect, which holds utility constant while changing and the income effect, which adjusts for the loss of purchasing power arising from the price increase.

Example (Cobb-Douglas): Recall that the Cobb-Douglas utility comes in the form $u(x, y) = x^{\alpha} y^{1-\alpha}$. Solving for x, y we obtain

$$x = \frac{\alpha M}{p_X}, \qquad y = \frac{(1-\alpha)M}{p_Y}$$

and

$$u(x, y) = \alpha^{\alpha}(1-\alpha)^{1-\alpha} \frac{M}{p_X^{\alpha} p_Y^{1-\alpha}}$$

Thus, consider a multiplicative increase Δ in pY; that is, multiplying pY by $\Delta > 1$. In order to leave the utility constant, M must rise by $\Delta^{1-\alpha}$. Thus, x rises by the factor $\Delta^{1-\alpha}$ and y falls by the factor $\Delta^{-\alpha} < 1$. This is the substitution effect.

What is the substitution effect of a small change in the price pY for any given utility function, not necessarily Cobb-Douglas? To address this question, it is helpful to introduce some notation. We will subscript the utility to indicate partial derivative; that is,

$$u_1 = \frac{\partial u}{\partial x}, \qquad u_2 = \frac{\partial u}{\partial y}$$

Note that, by the definition of the substitution effect, we are holding the utility constant, so $u(x, y)$ is being held constant. This means, locally, that $0 = du = u_1\, dx + u_2\, dy$.[4]

In addition, we have $M = p_X x + p_Y y$, so $dM = p_X dx + p_Y dy + y dp_Y$.

Finally, we have the optimality condition $\frac{p_X}{p_Y} = \frac{\partial u / \partial x}{\partial u / \partial y}$, that is convenient to write as $p_X u_2 = p_Y u_1$. Differentiating this equation, and letting

Compensated demand

Demand that exists when a change in price is accompanied by just enough additional income to keep utility the same.

Income effect

The effect on consumption of a change in income.

$$u_{11} = \frac{\partial^2 u}{(\partial x)^2}, \quad u_{12} = \frac{\partial^2 u}{\partial x \partial y}, \quad and\ u_{22} = \frac{\partial^2 u}{(\partial y)^2}$$

we have

$$p_X(u_{12}dx + u_{22}dy) = u_1\, dp_Y + p_Y(u_{11}dx + u_{12}dy).$$

For a given dpY, we now have three equations in three unknowns: dx, dy, and dM. However, dM only appears in one of the three. Thus, the effect of a price change on x and y can be solved by solving two equations: $0 = u_1\, dx + u_2\, dy$ and $p_X(u_{12}dx + u_{22}dy) = u_1\, dp_Y + p_Y(u_{11}dx + u_{12}dy)$ for the two unknowns, dx and dy. This is straightforward and yields:

$$\frac{dx}{dp_Y} = -\frac{p_Y u_1}{p_X^2 u_{11} + 2p_X p_Y u_{12} + p_Y^2 u_{22}} \text{ and}$$

$$\frac{dy}{dp_Y} = \frac{p_Y u_2}{p_X^2 u_{11} + 2p_X p_Y u_{12} + p_Y^2 u_{22}}$$

These equations imply that x rises and y falls.[5] We immediately see that

$$\frac{\frac{dy}{dp_Y}}{\frac{dx}{dp_Y}} = -\frac{u_1}{u_2} = -\frac{p_X}{p_Y}$$

Thus, the change in (x, y) follows the budget line locally. (This is purely a consequence of holding utility constant.)

To complete the thought while we are embroiled in these derivatives, note that $p_X u_2 = p_Y u_1$ implies that $p_X dx + p_Y dy = 0$.

Thus, the amount of money necessary to compensate the consumer for the price increase, keeping utility constant, can be calculated from our third equation:

$$dM = p_X dx + p_Y dy + y dp_Y = y dp_Y$$

The amount of income necessary to insure that the consumer makes no losses from a price increase in Y is the amount that lets him or her buy the bundle which he or she originally purchased; that is, the increase in the amount of money is precisely the amount needed to cover the increased price of y. This shows that locally there is no difference between a substitution effect that keeps utility constant (which is what we explored) and one that provides sufficient income to permit purchasing the previously purchased consumption bundle, at least when small changes in prices are contemplated.

KEY TAKEAWAYS

- An increase in the price of a good is really a composition of two effects: An increase in the relative price of the good, and a decrease in the purchasing power of money. It is useful to examine these two effects separately. The substitution effect considers the change in the relative price, with a sufficient change in income to keep the consumer on the same utility isoquant. The income effect changes only income.
- The substitution effect is the change in consumption resulting from a price change keeping utility constant. The substitution effect always involves a reduction in the good whose price increased.
- The amount of money required to keep the consumer's utility constant from an infinitesimal price increase is precisely the amount required to let him or her buy his or her old bundle at the new prices.

6. INCOME EFFECTS

LEARNING OBJECTIVE

1. How do consumers change their purchases when their income rises or falls?

Wealthy people buy more caviar than poor people. Wealthier people buy more land, medical services, cars, telephones, and computers than poorer people because they have more money to spend on goods and services, and overall buy more of them. But wealthier people also buy fewer of some goods, too. Rich people buy fewer cigarettes and processed cheese foods. You don't see billionaires waiting in line at McDonald's, and that probably isn't because they have an assistant to wait in line for them. For most goods, at a sufficiently high income, the purchase tends to trail off as income rises.

When an increase in income causes a consumer to buy more of a good, that good is called a normal good for that consumer. When the consumer buys less, the good is called an inferior good, which is an example of sensible jargon that is rare in any discipline. That is, an inferior good is any good whose quantity demanded falls as income rises. At a sufficiently low income, almost all goods are normal goods, while at a sufficiently high income, most goods become inferior. Even a Ferrari is an inferior good against some alternatives, such as Lear jets.

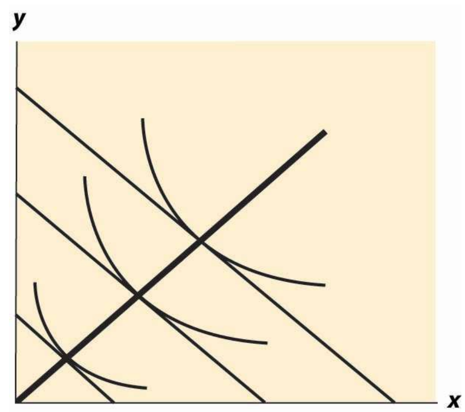
The curve that shows the path of consumption as income changes, holding prices constant, is known as an **Engel curve**.[6] An Engel curve graphs $(x(M), y(M))$ as M varies, where $x(M)$ is the amount of X chosen with income M, and similarly $y(M)$ is the amount of Y. An example of an Engel curve is illustrated in Figure 6.1.

Engel curve

Graph that shows the path of consumption as income changes, holding prices constant.

Example (Cobb-Douglas): Since the equations $x = \frac{\alpha M}{p_X}$, $y = \frac{(1-\alpha)M}{p_Y}$ define the optimal consumption, the Engel curve is a straight line through the origin with slope $\frac{(1-\alpha)p_X}{\alpha p_Y}$.

FIGURE 6.1 Engel curve



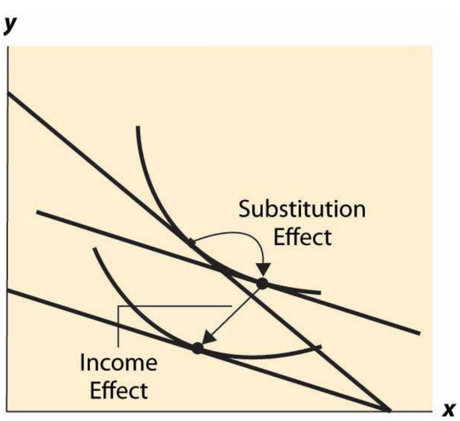
An inferior good will see the quantity fall as income rises. Note that, with two goods, at least one is a normal good—they can't both be inferior goods because otherwise, when income rises, less of both would be purchased. An example of an inferior good is illustrated in Figure 6.2. Here, as income rises, the consumption of x rises, reaches a maximum, and then begins to decline. In the declining portion, X is an inferior good.

The definition of the substitution effect now permits us to decompose the effect of a price change into a substitution effect and an income effect. This is illustrated in Figure 6.3.

What is the mathematical form of the income effect? This is actually more straightforward to compute than the substitution effect computed above. As with the substitution effect, we differentiate the conditions $M = p_x x + p_y y$ and $p_x u_2 = p_y u_1$, holding pX and pY constant, to obtain: $dM = p_X dx + p_Y dy$ and $p_X(u_{12}dx + u_{22}dy) = p_Y(u_{11}dx + u_{12}dy)$.

FIGURE 6.2 Backward bending—inferior good

FIGURE 6.3 Income and substitution effects



The second condition can also be written as $\frac{dy}{dx} = \frac{p_Y u_{11} - p_X u_{12}}{p_X u_{22} - p_Y u_{12}}$.

This equation alone defines the slope of the Engel curve without determining how large a change arises from a given change in M. The two conditions together can be solved for the effects of M on X and Y. The Engel curve is given by

$$\frac{dx}{dM} = \frac{p_Y^2 u_{11} - 2p_X u_{12} + p_X^2 u_{22}}{p_X u_{22} - p_Y u_{12}}$$

and

$$\frac{dy}{dM} = \frac{p_Y^2 u_{11} - 2p_X u_{12} + p_X^2 u_{22}}{p_Y u_{11} - p_X u_{12}}$$

Note (from the second-order condition) that good Y is inferior if $p_Y u_{11} - p_X u_{12} > 0$, or if $\frac{u_{11}}{u_1} - \frac{u_{12}}{u_2} > 0$, or $\frac{u_1}{u_2}$ is increasing in x. Since $\frac{u_1}{u_2}$ is locally constant when M increases, equaling the price ratio, and an increase in y increases $\frac{u_1}{u_2}$ (thanks to the second-order condition), the only way to keep $\frac{u_1}{u_2}$ equal to the price ratio is for x to fall. This property characterizes an inferior good—an increase in the quantity of the good increases the marginal rate of substitution of that good for another good.

KEY TAKEAWAYS

- When an increase in income causes a consumer to buy more of a good, that good is called a normal good for that consumer. When the consumer buys less, the good is called an inferior good. At a sufficiently high income, most goods become inferior.
- The curve that shows the path of consumption as income rises is known as an Engel curve.
- For the Cobb-Douglas utility, Engel curves are straight lines through the origin.
- Not all goods can be inferior.
- The effect of a price increase decomposes into two effects—a decrease in real income and a substitution effect from the change in the price ratio. For normal goods, a price increase decreases quantity. For inferior goods, a price increase decreases quantity only if the substitution effect is larger than the income effect.